UCRHEP-T453
August 2008

Dark Scalar Doublets and Neutrino Tribimaximal Mixing from A_4 Symmetry

Ernest Ma

Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

Abstract

In the context of A_4 symmetry, neutrino tribimaximal mixing is achieved through the breaking of A_4 to Z_3 (Z_2) in the charged-lepton (neutrino) sector respectively. The implied vacuum misalignment of the (1,1,1) and (1,0,0) directions in A_4 space is a difficult technical problem, and cannot be treated without many auxiliary fields and symmetries (and perhaps extra dimensions). It is pointed out here that an alternative scenario exists with A_4 alone and no redundant fields, if neutrino masses are "scotogenic", i.e. radiatively induced by dark scalar doublets as recently proposed.

The neutrino mixing angles are now known to some accuracy. Based on a recent global analysis [1],

$$\theta_{23} = 42.3\ (+5.1/-3.3), \quad \theta_{12} = 34.5 \pm 1.4, \quad \theta_{13} = 0.0\ (+7.9/-0.0), \tag{1}$$

at the 1σ level. Thus the central values of $\sin^2 2\theta_{23}$, $\tan^2\theta_{12}$, and θ_{13} are 0.99, 0.47, and 0 respectively. These numbers agree well with the hypothesis of tribimaximal mixing [2], i.e.

$$\sin^2 2\theta_{23} = 1, \quad \tan^2\theta_{12} = 0.5, \quad \theta_{13} = 0. \tag{2}$$

Such a pattern is best understood as the result of a family symmetry and the non-Abelian finite group A_4 has proved to be useful in this regard [3, 4, 5]. Specifically, it was shown [6, 7, 8] how this may be achieved by the breaking of A_4 in a prescribed manner [9], i.e. $A_4 \to Z_3$ in the charged-lepton sector and $A_4 \to Z_2$ in the neutrino sector. The group-theoretical framework of how this works in general has also been discussed [10, 11]. For a brief history, see Ref. [12].

In another development, it has been proposed recently [13] that neutrino mass is radiative in origin such that the particles in the loop are odd under a new discrete Z_2' symmetry, thereby accommodating a dark-matter candidate. The simplest realization of this "scotogenic" neutrino mass is depicted in Fig. 1. Here N_k are heavy Majorana fermion singlets

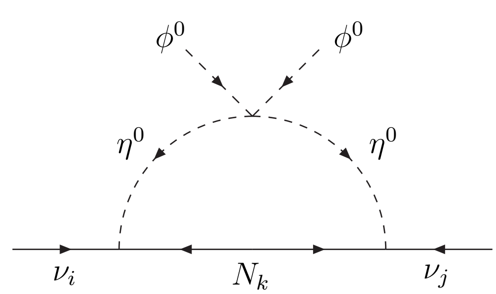

Figure 1: One-loop generation of seesaw neutrino mass.

odd under Z_2' and (η^+, η^0) is a scalar doublet also odd under Z_2' [14], whereas the standard-model (ϕ^+, ϕ^0) is even. Exact conservation of Z_2' means of course that η^0 has no vacuum expectation value, so that N is not the Dirac mass partner of ν as usually assumed. The allowed quartic coupling $(\lambda_5/2)(\Phi^\dagger \eta)^2 + H.c.$ splits $\mathrm{Re}(\eta^0)$ and $\mathrm{Im}(\eta^0)$ so that whichever is lighter is a good dark-matter candidate [13, 15, 16, 17]. The collider signatures of $\mathrm{Re}(\eta^0)$ and $\mathrm{Im}(\eta^0)$ have also been discussed [18]. For a brief review of the further developments of this idea, see Ref. [19].

Going back to A_4, let $(\nu_i, l_i) \sim \underline{3}$ and either (I) $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, or (II) $l_i^c \sim \underline{3}$, then with the Higgs fields (I) $(\phi_i^+, \phi_i^0) \sim \underline{3}$, or (II) $(\phi_i^+, \phi_i^0) \sim \underline{3}$ and $(\zeta^+, \zeta^0) \sim \underline{1}$, the mass matrix linking l_i to l_j^c is diagonalized on the left by [9]

$$U_{l\nu} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{3}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, if $\langle \phi_1^0 \rangle = \langle \phi_2^0 \rangle = \langle \phi_3^0 \rangle = v$. This is a natural minimum of the Higgs potential [3] because it corresponds to a Z_3 residual symmetry with $e \sim 1$, $\mu \sim \omega^2$, $\tau \sim \omega$, whereas $\Phi \equiv (\Phi_1 + \Phi_2 + \Phi_3)/\sqrt{3} \sim 1$, $\Phi' \equiv (\Phi_1 + \omega \Phi_2 + \omega^2 \Phi_3)/\sqrt{3} \sim \omega^2$, and $\Phi'' \equiv (\Phi_1 + \omega^2 \Phi_2 + \omega \Phi_3)/\sqrt{3} \sim \omega$.

To obtain tribimaximal mixing, what is required for the Majorana neutrino mass matrix $\mathcal{M}_\nu$ is [6] $2-3$ symmetry and zero $1-2$ and $1-3$ entries. Since $123 + 231 + 312$ and $132 + 321 + 213$ are A_4 invariants and $\mathcal{M}_\nu$ must be symmetric, the simplest implementation is to have [7]

$$\mathcal{M}_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}, \tag{4}$$

which requires effective scalar triplet fields $(\xi_i^{++}, \xi_i^+, \xi_i^0)$ transforming as $\underline{3}$ with $\langle \xi_1^0 \rangle \neq 0$ and $\langle \xi_{2,3}^0 \rangle = 0$, thereby breaking $A_4 \to Z_2$. Let the eigenvalues of $\mathcal{M}_\nu$ be denoted by

$$m_1 = a + d, \quad m_2 = a, \quad m_3 = -a + d, \tag{5}$$

then the mixing matrix linking $\nu_{e,\mu,\tau}$ to $\nu_{1,2,3}$ is given by [12]

$$(U_{l\nu})^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \qquad (6)$$

i.e. tribimaximal mixing.

Because the scalar fields Φ_i and ξ_i are both $\underline{3}$ under A_4, the requirement that they break the vacuum in different directions is incompatible with the most general Higgs potential allowed by A_4 alone. Complicated sets of auxiliary fields and symmetries (and/or possible extra dimensions) are then needed [7, 8, 20, 21, 22] for it to happen. This is perhaps the one stumbling block of the application of A_4 to tribimaximal mixing.

The reason that the two breaking directions are incompatible is because A_4 allows $\underline{3} \times \underline{3}$ to be invariant, so if one $\underline{3}$ has a vacuum expectation value along a certain direction, the other is forced to as well. This is of course not a problem if only one $\underline{3}$ is required to have vacuum expectation values and not the other, because that corresponds to having an exactly conserved Z_2' under which the second $\underline{3}$ is odd. Specifically, let the charged leptons acquire mass from Φ_i, but the neutrino masses are obtained radiatively as discussed earlier, without any vacuum expectation value for η^0.

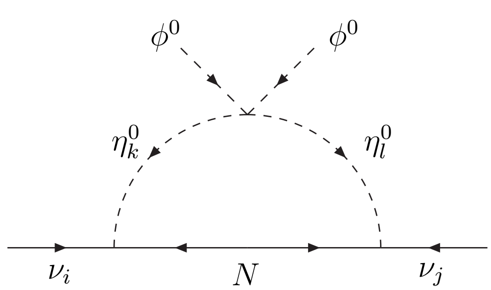

Figure 2: One-loop generation of seesaw neutrino mass.

Instead of having three N's (which would have been necessary in the canonical seesaw mechanism), assume just one N but three scalar η doublets, as shown in Fig. 2. Let (η_i^+, η_i^0)

transform as $\underline{3}$ under A_4, then $\mathcal{M}_\nu$ is proportional to the unit matrix. Suppose A_4 is now softly broken by the quadratic terms $\eta_2^\dagger\eta_3 + \eta_3^\dagger\eta_2$ and $2\eta_1^\dagger\eta_1 - \eta_2^\dagger\eta_2 - \eta_3^\dagger\eta_3$. Then $\mathcal{M}_\nu$ is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b & 0 & 0 \\ 0 & a-b & d \\ 0 & d & a-b \end{pmatrix}, \tag{7}$$

which will lead to tribimaximal mixing [6], with

$$m_1 = a - b + d, \quad m_2 = a + 2b, \quad m_3 = -a + b + d. \tag{8}$$

Since the origin of $\mathcal{M}_\nu$ is the mass-squared matrix of $\eta^0_{1,2,3}$, this model may be tested at least in principle. Note that $b = 0$ cannot be a solution here as in Ref. [7] because that would require a negative mass-squared eigenvalue for η_i^0. As it is, $\Delta m^2_{sol} << \Delta m^2_{atm}$ implies $d \simeq 3b$ or $-2a - b$ in this scenario.

Consider now the scalar sector in more detail. Since η_i are odd under the new exactly conserved Z_2' for dark matter, and have no vacuum expectation value. The bilinear terms $\Phi_i^\dagger\eta_j$ are forbidden, and the quartic terms must contain an even number of Φ_i and η_j. The scalar potential consisting of only Φ_i is given by [3]

$$\begin{aligned} V_\Phi &= m^2 \sum_i \Phi_i^\dagger\Phi_i + \frac{1}{2}\lambda_1 \left(\sum_i \Phi_i^\dagger\Phi_i\right)^2 \\ &+ \lambda_2(\Phi_1^\dagger\Phi_1 + \omega^2\Phi_2^\dagger\Phi_2 + \omega\Phi_3^\dagger\Phi_3)(\Phi_1^\dagger\Phi_1 + \omega\Phi_2^\dagger\Phi_2 + \omega^2\Phi_3^\dagger\Phi_3) \\ &+ \lambda_3[(\Phi_2^\dagger\Phi_3)(\Phi_3^\dagger\Phi_2) + (\Phi_3^\dagger\Phi_1)(\Phi_1^\dagger\Phi_3) + (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ \left\{\frac{1}{2}\lambda_4[(\Phi_2^\dagger\Phi_3)^2 + (\Phi_3^\dagger\Phi_1)^2 + (\Phi_1^\dagger\Phi_2)^2] + H.c.\right\}. \end{aligned} \tag{9}$$

The parameters m^2 and $\lambda_{1,2,3}$ are automatically real, and λ_4 may be chosen real by rotating the overall phase of Φ_i. The vacuum solution

$$v_1 = v_2 = v_3 = v = [-m^2/(3\lambda_1 + 2\lambda_3 + 2\lambda_4)]^{1/2} \tag{10}$$

is protected by the residual symmetry Z_3, under which

$$\Phi \equiv \frac{1}{\sqrt{3}}(\Phi_1 + \Phi_2 + \Phi_3) \sim 1 \tag{11}$$

$$\Phi' \equiv \frac{1}{\sqrt{3}}(\Phi_1 + \omega\Phi_2 + \omega^2\Phi_3) \sim \omega^2, \tag{12}$$

$$\Phi'' \equiv \frac{1}{\sqrt{3}}(\Phi_1 + \omega^2\Phi_2 + \omega\Phi_3) \sim \omega, \tag{13}$$

as already mentioned. The scalar doublet Φ has the properties of the standard-model Higgs doublet with mass-squared eigenvalues $2(3\lambda_1 + 2\lambda_3 + \lambda_4)v^2$, 0, and 0 for $\sqrt{2}\mathrm{Re}\phi^0$, $\sqrt{2}\mathrm{Im}\phi^0$, and $\phi^\pm$ respectively. The charged scalars $\phi'^\pm$ and $\phi''^\pm$ have $m^2_\pm = -3(\lambda_3+\lambda_4)v^2$, whereas ϕ'^0 and ϕ''^0 are not mass eigenstates, but rather $\phi'^0 = (\psi_1 + \psi_2)/\sqrt{2}$ and $\phi''^0 = (\psi_1^* - \psi_2^*)/\sqrt{2}$, i.e.

$$\psi_1 = \frac{1}{\sqrt{2}}\mathrm{Re}(\phi'^0 + \phi''^0) + \frac{i}{\sqrt{2}}\mathrm{Im}(\phi'^0 - \phi''^0) \sim \omega^2, \tag{14}$$

$$\psi_2 = \frac{1}{\sqrt{2}}\mathrm{Re}(\phi'^0 - \phi''^0) + \frac{i}{\sqrt{2}}\mathrm{Im}(\phi'^0 + \phi''^0) \sim \omega^2, \tag{15}$$

with $m_1^2 = 2(3\lambda_2 - \lambda_3 - \lambda_4)v^2$ and $m_2^2 = -6\lambda_4 v^2$. This subtlety in the mass spectrum of ϕ'^0 and ϕ''^0 was not recognized in Ref. [3], where $\tau^- \to \mu^-\mu^+e^-$ and $\mu \to e\gamma$ were thought to be nonzero. In fact, they are forbidden by the residual Z_3 symmetry.

The addition of η_i to the scalar potential does not change the above because $\langle \eta_i^0 \rangle = 0$ and Z_2' remains exactly conserved. However, the breaking of $A_4 \to Z_3$ by $\langle \phi_i^0 \rangle$ generates additional contributions to the η_i^0 mass-squared matrix of the form

$$\begin{aligned}&\Delta_1^2(\eta_1^*\eta_1 + \eta_2^*\eta_2 + \eta_3^*\eta_3) + \{\Delta_2^2(\eta_1^*\eta_2 + \eta_2^*\eta_3 + \eta_3^*\eta_1) + c.c.\} \\ &+\{\frac{1}{2}\Delta_3^2(\eta_1^2 + \eta_2^2 + \eta_3^2) + c.c.\} + \{\Delta_4^2(\eta_1\eta_2 + \eta_2\eta_3 + \eta_3\eta_1) + c.c.\}.\end{aligned} \tag{16}$$

In other words, except for soft terms, the complete Higgs potential remains invariant under Z_3 after spontaneous symmetry breaking. The induced neutrino mass matrix of Eq. (7) is then modified:

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b & e & e \\ e & a-b & d \\ e & d & a-b \end{pmatrix}. \tag{17}$$

Since the one-loop neutrino mass of Fig. 1 is proportional to Δ_3^2 and Δ_4^2 which split $\mathrm{Re}(\eta_i^0)$ and $\mathrm{Im}(\eta_i^0)$, these parameters should be relatively small. Assuming that Δ_2^2 is also small,

then e should be small compared to a, b, d in Eq. (17). This means that [23] $\sin^2 2\theta_{23} = 1$ and $\theta_{13} = 0$ as before, but the solar mixing angle is now given by

$$\tan^2 \theta_{12} = \frac{1}{2}(1 - 6\epsilon + 15\epsilon^2), \tag{18}$$

where $\epsilon = e/(d - 3b)$. Thus $\tan^2 \theta_{12} = 0.47$ is obtained for $\epsilon = 0.01$.

One possible explanation of the smallness of the terms in Eq. (16) is that Φ and η are separated in an extra dimension so that they communicate only through a singlet in the bulk. In the limit this effect vanishes, there would be no mass splitting between $\mathrm{Re}(\eta^0)$ and $\mathrm{Im}(\eta^0)$, resulting in zero neutrino mass and no viable dark-matter candidate. With it, neutrinos acquire small radiative Majorana seesaw masses, $\mathrm{Re}(\eta^0)$ is a good dark-matter candidate, and near tribimaximal mixing is possible.

In conclusion, it has been shown how A_4 symmetry may be implemented in a model of "scotogenic" neutrino mass with dark scalar doublets. The neutrino mass matrix is induced by the neutral scalar mass-squared matrix spanning $\mathrm{Re}(\eta^0_{1,2,3})$ and $\mathrm{Im}(\eta^0_{1,2,3})$. This scheme allows the neutrino mixing angles θ_{23} and θ_{13} to be exactly $\pi/4$ and 0, whereas $\tan^2 \theta_{12}$ should not be exactly 1/2. Suppose the lightest $\mathrm{Re}(\eta^0)$ is dark matter, then its possible discovery [18] at the LHC together with the other η particles in accordance with Fig. 2 would be a verifiable test of this proposal.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

References

[1] M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rep. **460**, 1 (2008).

[2] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. **B530**, 167 (2002).

[3] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001).

[4] E. Ma, Mod. Phys. Lett. **A17**, 2361 (2002).

[5] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).

[6] E. Ma, Phys. Rev. **D70**, 031901(R) (2004).

[7] G. Altarelli and F. Feruglio, Nucl. Phys. **B720**, 64 (2005).

[8] K. S. Babu and X.-G. He, hep-ph/0507217.

[9] E. Ma, Mod. Phys. Lett. **A21**, 2931 (2006).

[10] C. S. Lam, Phys. Lett. **B656**, 193 (2007).

[11] A. Blum, C. Hagedorn, and M. Lindner, Phys. Rev. **D77**, 076004 (2008).

[12] E. Ma, arXiv:0710.3851 [hep-ph].

[13] E. Ma, Phys. Rev. **D73**, 077301 (2006).

[14] N. G. Deshpande and E. Ma, Phys. Rev. **D18**, 2574 (1978).

[15] R. Barbieri, L. J. Hall, and V. S. Rychkov, Phys. Rev. **D74**, 015007 (2006).

[16] L. Lopez Honorez, E. Nezri, J. F. Oliver, and M. H. G. Tytgat, JCAP **02**, 028 (2007).

[17] M. Gustafsson, E. Lundstrom, L. Bergstrom, and J. Edsjo, Phys. Rev. Lett. **99**, 041301 (2007).

[18] Q.-H. Cao, E. Ma, and G. Rajasekaran, Phys. Rev. **D76**, 095011 (2007).

[19] E. Ma, Mod. Phys. Lett. **A23**, 647 (2008).

[20] G. Altarelli and F. Feruglio, Nucl. Phys. **B741**, 215 (2006).

[21] X.-G. He, Nucl. Phys. Proc. Suppl. **168**, 350 (2007).

[22] C. Csaki, C. Delaunay, C. Grojean, and Y. Grossman, arXiv:0806.0356 [hep-ph].

[23] E. Ma, Phys. Rev. **D66**, 117301 (2002).